Quantitative Forecasting Methods

- Naïve Approach
- Moving Averages
- Exponential Smoothing
- Trend projection
- *Linear Regression*

Time-Series Model

Associative Model

Exponential Smoothing Method

- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant (α)
  - Ranges from 0 to 1
  - Subjectively chosen
- Involves little record keeping of past data

Exponential Smoothing

New forecast =

previous forecast + ~(previous actual - previous)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

or:

where

F_t = new forecast

F_{t-1} = previous forecast

α = smoothing constant

A_{t-1} = previous period actual

Exponential Smoothing Equations

- $F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 \cdot A_{t-3} + \alpha(1-\alpha)^3 A_{t-4} + \ldots + \alpha(1-\alpha)^{t-1} \cdot A_0$
  - F_t = Forecast value
  - A_t = Actual value
  - α = Smoothing constant

Forecast Effects of Smoothing Constant α

$$F_t = \sim A_{t-1} + \sim(1 - \sim) A_{t-2} + \sim(1 - \sim)^2 A_{t-3} + \ldots$$

~=	Weights		
	Prior Period	2 periods ago	3 periods ago
	$\sim$	$\sim(1 - \sim)$	$\sim(1 - \sim)^2$
~= 0.10	10%	9%	8.1%
~= 0.90	90%	9%	0.9%

Table 5.4

TABLE 5.4

Port of Baltimore Exponential Smoothing Forecasts for $\alpha = 0.10$ and $\alpha = 0.50$

QUARTER	ACTUAL TONNAGE UNLOADED	ROUNDED FORECAST USING $\alpha = 0.10$*	ROUNDED FORECAST USING $\alpha = 0.50$*
1	180	175	175
2	168	176 = 175.00 + 0.10(180 – 175)	178
3	159	175 = 175.50 + 0.10(168 – 175.50)	173
4	175	173 = 174.75 + 0.10(159 – 174.75)	166
5	190	173 = 173.18 +0 .10(175 – 173.18)	170
6	205	175 = 173.36 + 0.10(190 – 173.36)	180
7	180	178 = 175.02 + 0.10(205 – 175.02)	193
8	182	178 = 178.02 + 0.10(180 – 178.02)	186
9	?	179 = 178.22 + 0.10(182 – 178.22)	184

* Forecasts rounded to the nearest ton.

Exponential Smoothing with Trend Adjustment

- **Simple exponential smoothing - *first-order smoothing***
- **Trend adjusted smoothing - *second-order smoothing***

- **Low   gives less weight to more recent trends, while high   gives higher weight to more recent trends**

Selecting the Smoothing Constant (α)

Select ~ to minimize:

Mean Absolute Deviation = MAD $= \dfrac{\Sigma \mid \text{forecast errors} \mid}{n}$

Mean Square Error = MSE $= \dfrac{\Sigma(\text{forecast errors})^2}{n}$

Mean Absolute Percent Error = MAPE

$$= \frac{1}{n} \Sigma \left[\frac{\left|\text{forecast error}\right|}{\text{actual}} \right]$$

$$\text{Bias} = \sum \text{forecast errors}$$

Error Analysis (Alpha=0.1)

Details and Error Analysis

(untitled) Solution						
	Demand(y)	Forecast	Error	|Error|	Error^2	|Pct Error|
January	180					
February	168	180	-12	12	144	.07
March	159	178.8	-19.8	19.8	392.04	.12
April	175	176.82	-1.82	1.82	3.31	.01
May	190	176.64	13.36	13.36	178.54	.07
June	205	177.97	27.03	27.03	730.39	.13
July	180	180.68	-.68	.68	.46	0
August	182	180.61	1.39	1.39	1.93	0
TOTALS	1439		7.48	76.08	1450.68	.42
AVERAGE	179.88		1.07	10.87	207.24	.06
Next period forecast		180.75	(Bias)	(MAD)	(MSE)	(MAPE)
				Std err	17.03	

Error Analysis (Alpha=0.5)

	Demand(y)	Forecast	Error	|Error|	Error^2	|Pct Error|
January	180					
February	168	180	-12	12	144	.07
March	159	174	-15	15	225	.09
April	175	166.5	8.5	8.5	72.25	.05
May	190	170.75	19.25	19.25	370.56	.1
June	205	180.38	24.63	24.63	606.39	.12
July	180	192.69	-12.69	12.69	160.97	.07
August	182	186.34	-4.34	4.34	18.87	.02
TOTALS	1439		8.34	96.41	1598.04	.53
AVERAGE	179.88		1.19	13.77	228.29	.08
Next period forecast		184.17	(Bias)	(MAD)	(MSE)	(MAPE)
				Std err	17.88	

Exponential Smoothing with Trend Adjustment

- **Simple exponential smoothing - *first-order smoothing***
- **Trend adjusted smoothing - *second-order smoothing***

- **Low   gives less weight to more recent trends, while high   gives higher weight to more recent trends**

Exponential Smoothing with Trend Adjustment

Forecast including trend (FIT_{t+1}) = new forecast (F_t) + trend correction(T_t)

$$T_t = (1 - \beta)T_{t-1} + \beta(F_t - F_{t-1})$$

where

T_t = smoothed trend for period t

T_{t-1} = smoothed trend for the preceding period

β = trend smoothing constant

F_t = simple exponential smoothed forecast for period t

F_{t-1} = forecast for period t-1

Exponential Forecast with TREND (Alpha=0.1, Beta=0.5)

	Demand(y)	unadjusted forecast	trend	adjusted forecast	error	|Error|	Error^2	|Pct Error|
January	180							
February	168	178.8	-.6	180	-12	12	144	.07
March	159	176.28	-1.56	178.2	-19.2	19.2	368.64	.12
April	175	174.75	-1.55	174.72	.28	.28	.08	0
May	190	174.88	-.71	173.2	16.8	16.8	282.17	.09
June	205	177.26	.84	174.18	30.82	30.82	950.14	.15
July	180	178.28	.93	178.09	1.91	1.91	3.64	.01
August	182	179.49	1.07	179.21	2.79	2.79	7.76	.02
TOTALS	1439				21.39	83.79	1756.43	.46
AVERAGE	179.88				3.06	11.97	250.92	.07
Next period forecast				180.56	(Bias)	(MAD)	(MSE)	(MAPE)
						Std err	18.74	

Trend Projection

General regression equation:

$$\hat{Y} = a + bX$$

where

$\hat{Y}$ = computed value of the variable to be predicted (dependent variable)

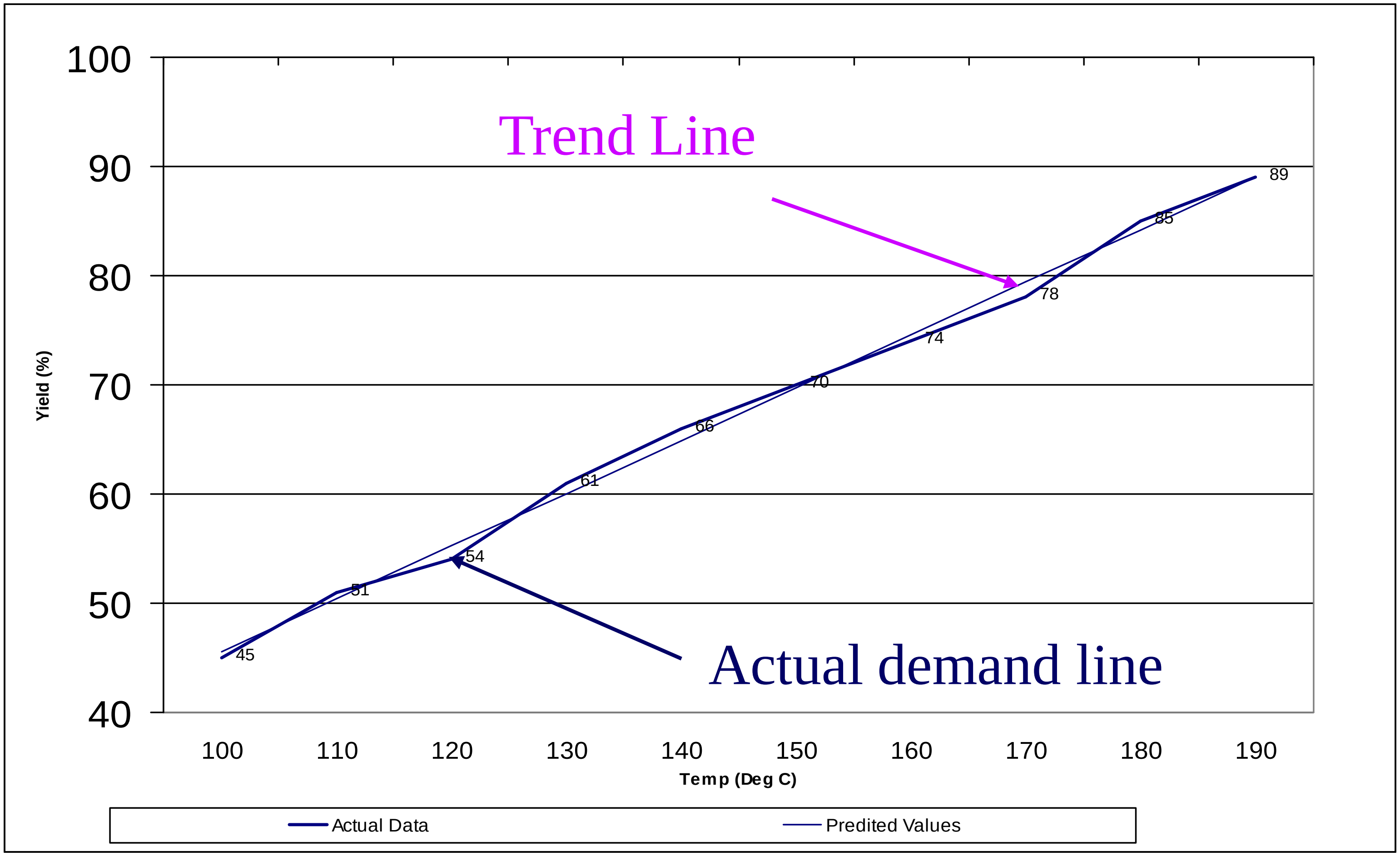
a = Y - axis intercept

$$b = \frac{\Sigma XY - nXY}{\Sigma X^2 - nX^2}$$

Trend Projections

Data No	Temperature Deg C (X)	Yield % (Y)	X*X	XY	Y*Y	Forecast Value		n=	10
1	100	45	10000	4500	2025	45.5636		sumX=	1450
2	110	51	12100	5610	2601	50.3939		sumY=	673
3	120	54	14400	6480	2916	55.2242		sumX*X=	218500
4	130	61	16900	7930	3721	60.0545		sumX*Y=	101570
5	140	66	19600	9240	4356	64.8848		sumY*Y=	47225
6	150	70	22500	10500	4900	69.7152		aveX=	145
7	160	74	25600	11840	5476	74.5455		aveY=	67.3
8	170	78	28900	13260	6084	79.3758		STD_ERR_EST	82.42589
9	180	85	32400	15300	7225	84.2061		R_SQR	0.996261
10	190	89	36100	16910	7921	89.0364		intercept(a)=	-2.739394
								slope(b)=	0.48303

Graph: Actual vs Fitted Line



Seasonal Variations

Month	Sales Demand		Average Two-Year Demand	Average Monthly Demand	Seasonal Index
	Year 1	Year 2			
Jan	80	100	90	94	0.957
Feb	75	85	80	94	0.851
Mar	80	90	85	94	0.904
Apr	90	110	100	94	1.064
May	115	131	123	94	1.309
...	...	...	...	...	...

1128/12

Total of Average Demand = 1,128

Seasonal Index:

= Average 2 -year demand/Average monthly demand

If total demand/year for year 3=1200, then monthly forecast= (1200/12)* seasonal index

Month	Sales Demand		Average Two-Year Demand	Average Monthly Demand	Seasonal Index
	Year 1	Year 2			
Jan	80	100	90	94	0.957
Feb	75	85	80	94	0.851
Mar	80	90	85	94	0.904
Apr	90	110	100	94	1.064
May	115	131	123	94	1.309
...	...	...	...	...	...

95.7
85.1
90.4
106.4
130.9

1128/12

Total of Average Demand = 1,128
Seasonal Index:

= Average 2 -year demand/Average monthly demand